2 Mathematics

Program Library

Algebra
Calculus
Geometry
Trigonometry
Number Theory
Transcendental Functions

Sinclair Cambridge Programmable

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Mathematics

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CONTENTS

How to use these programs

Each program is arranged as follows:

1. On the left of the page, explanatory information and the 'execution sequence', the sequence of keystrokes necessary for running the program. Results displayed are printed in gold.
2. In the first column on the right hand side of the page, the sequence of keystrokes which make up the program.
3. In the second and third columns on the right hand side of the page, the program in check symbol and step number form (see section on checking the program).

Notes

1. Where a key has more than one function, the relevant function is printed as the keystroke in the first column

   e.g. the keystroke [8] (cos / arccos) may appear as 8, cos or arccos.
2. The symbol ▾ within a program always refers to the key [·/EE/–] (▾)
3. The symbol # refers to [3] (ChN/#)
4. The abbreviation gin is 'go if neg' and so refers to the key [1] (go if neg)

Entering the program

To enter a program into the calculator:

1. Press [▲▼] [▲▼] [2] (go to) [0] [0]	Display shows step programmed at 00 in check symbol form as described below.
2. Press [▲▼] [RUN] (learn)	No change in display.
3. Press the sequence of keys for the program as shown in the first column of the program page.	At each stage the step about to be overwritten is displayed. When the machine is first switched on every step is zero.
4. Press [C/CE]	Normal number display is resumed.
5. Press [▲▼] [▲▼] [2] (go to) [0] [0]	The step programmed at 00 will be displayed.

Checking the program

Each of the programs in the library is shown in check symbol form in the second column on the right-hand side of the page.

Press [▲▼] [C/CE] (step) repeatedly, and at each stage the check symbol will appear on the left of the display with the step number on the right. Ignore the four zeros in the display.

e.g. A.0000 03 (check symbol: A; step number: 03)

After stepping through the program, press

[▲▼] [▲▼] [2] (go to) [0] [0] before execution.

Finally, press [C/CE] and the program is ready for use.

Correcting the program

If the check symbol for a particular step number is not as indicated in the last two columns of the program page:

1. Press [▲▼] [▲▼] [2] (go to) followed by the step number if the appropriate step number is not already displayed.
2. Press [▲▼] [RUN] (learn)
3. Enter the correct keystroke. The display will then show the next step in the program. If this is also incorrect, enter the correct keystroke. At each stage, the step about to be overwritten will be displayed.
4. When correction has been completed, press [C/CE]. Any step which has not been overwritten will not be affected.
5. Press [▲▼] [▲▼] [2] (go to) [0] [0]

Note
To restore normal use of the calculator after entering or checking the program, press [C/CE]

Running the program

Press the sequence of keys as shown in the program library in the execution sequence. Results displayed are printed in gold.

EXTENSION OF RANGE OF TRIGONOMETRIC FUNCTIONS

to $-\pi < \theta < \pi$

Sine of any angle:

$$\sin\theta = \frac{2t}{1+t^2} \quad \text{where } t = \tan\frac{\theta}{2}$$

Execution:

θ / RUN / $\sin\theta$

For θ in degrees, insert / ▼ / D→R / at start of program.

÷	G	00
#	3	01
2	2	02
=	—	03
tan	9	04
÷	G	05
(	6	06
X	·	07
+	E	08
#	3	09
1	1	10
=	—	11
)	6	12
+	E	13
=	—	14
stop	0	15
▼	A	16
goto	2	17
0	0	18
0	0	19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

EXTENSION OF RANGE OF TRIGONOMETRIC FUNCTIONS

to $-\pi < \theta < \pi$

Cosine of any angle

$$\cos\theta = \frac{1-t^2}{1+t^2} \quad \text{where } t = \tan\frac{\theta}{2}$$

Execution:

θ / RUN / $\cos\theta$

÷	G	00
#	3	01
2	2	02
=	–	03
tan	9	04
×	·	05
+	E	06
#	3	07
1	1	08
÷	G	09
+	E	10
–	F	11
#	3	12
1	1	13
=	–	14
stop	0	15
▼	A	16
goto	2	17
0	0	18
0	0	19
		20
		21
		22
		23
		24
		25
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		29
		30
		31
		32
		33
		34
		35

EXTENSION OF RANGE OF TRIGONOMETRIC FUNCTIONS

to $-\pi < \theta < \pi$

Tangent of any angle

$$\tan\theta = \frac{2t}{1-t^2} \quad \text{where } t = \tan\frac{\theta}{2}$$

Execution:

θ / RUN / tan θ

÷	G	00
#	3	01
2	2	02
=	–	03
tan	9	04
÷	G	05
(	6	06
X	·	07
–	F	08
#	3	09
1	1	10
–	F	11
)	6	12
+	E	13
=	–	14
stop	0	15
▼	A	16
goto	2	17
0	0	18
0	0	19
		20
		21
		22
		23
		24
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		26
		27
		28
		29
		30
		31
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		33
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EXTENSION OF RANGE OF TRIGONOMETRIC FUNCTIONS

to $-\pi < \theta < \pi$

sin, cos and tan using t = $\tan\frac{\theta}{2}$

Execution:

θ / RUN / sin θ / RUN / cos θ / RUN / tan θ

÷	G	00
#	3	01
2	2	02
=	–	03
tan	9	04
sto	2	05
X	·	06
+	E	07
#	3	08
1	1	09
÷	G	10
=	–	11
▼	A	12
MEx	5	13
X	·	14
rcl	5	15
+	E	16
=	–	17
stop	0	18
▼	A	19
MEx	5	20
+	E	21
–	F	22
#	3	23
1	1	24
÷	G	25
stop	0	26
X	·	27
rcl	5	28
=	–	29
stop	0	30
▼	A	31
goto	2	32
0	0	33
0	0	34
		35

SINE AND COSINE OF ANY ANGLE

Sin: use program on right

Execution:

angle in degrees / RUN / sine

For radians version of program, insert / ▼ / R→D / at beginning and omit / = / = / at end.

Cos: *either* use program on right and execute by / ▲▼ / ▲▼ / goto / 0 / 4 / angle in degrees / RUN / cosine

or omit first four keystrokes of program on right and fill the empty spaces at the end with repeated / = / and execute by

angle in degrees / RUN / cosine

For radians version of program, insert / ▼ / R→D / at the beginning.

Note: E can appear if reduced angles > 1·57 radians.

–	F	00
#	3	01
9	9	02
0	0	03
X	·	04
=	–	05
$\sqrt{x}$	1	06
–	F	07
+	E	08
#	3	09
3	3	10
6	6	11
0	0	12
–	F	13
▼	A	14
gin	1	15
0	0	16
7	7	17
#	3	18
1	1	19
8	8	20
0	0	21
X	·	22
=	–	23
$\sqrt{x}$	1	24
–	F	25
#	3	26
9	9	27
0	0	28
=	–	29
▼	A	30
D→R	3	31
sin	7	32
stop	0	33
=	–	34
=	–	35

TANGENT OF ANY ANGLE

+	E	00
#	3	01
9	9	02
0	0	03
÷	G	04
(	6	05
X	·	06
=	–	07
$\sqrt{x}$	1	08
sto	2	09
)	6	10
–	F	11
X	·	12
(	6	13
rcl	5	14
–	F	15
+	E	16
#	3	17
1	1	18
8	8	19
0	0	20
–	F	21
▼	A	22
gin	1	23
1	1	24
5	5	25
#	3	26
9	9	27
0	0	28
=	–	29
▼	A	30
D→R	3	31
tan	9	32
)	6	33
=	–	34
stop	0	35

Execution:

angle in degrees / RUN / tangent

Note: E can appear if reduced angle > 1·57 radians.

HYPERBOLIC FUNCTIONS

▼	A	00
e^x	4	01
–	F	02
(	6	03
÷	G	04
)	6	05
÷	G	06
#	3	07
2	2	08
=	–	09
stop	0	10
▼	A	11
goto	2	12
0	0	13
0	0	14
		15
		16
		17
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		19
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If all the hyperbolic functions are likely to be required, use the 'gudermannian' program on page 21 . For the individual functions, the following can be used:

Sinh x

Execution:

x / RUN / sinh x

Range:

$-227{\cdot}95 \leqslant x \leqslant 230{\cdot}25$

(out-of-range may give wrong result without E)

HYPERBOLIC FUNCTIONS

Cosh x

Execution:

x / RUN / cosh x

Range:

$-227{\cdot}95 \leqslant x \leqslant 230{\cdot}25$

(out-of-range may give wrong result without E)

▼	A	00
e^x	4	01
+	E	02
(	6	03
÷	G	04
)	6	05
÷	G	06
#	3	07
2	2	08
=	–	09
stop	0	10
▼	A	11
goto	2	12
0	0	13
0	0	14
		15
		16
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HYPERBOLIC FUNCTIONS

Tanh x

Execution:

x / RUN / tanh x

Range:

$|x| \leqslant 113{\cdot}97$

(out-of-range may give wrong result without E)

+	E	00
=	–	01
▼	A	02
e^x	4	03
+	E	04
#	3	05
1	1	06
÷	G	07
+	E	08
–	F	09
#	3	10
1	1	11
–	F	12
=	–	13
stop	0	14
▼	A	15
goto	2	16
0	0	17
0	0	18
		19
		20
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HYPERBOLIC FUNCTIONS

Sech x

Execution:

x / RUN / [illegible]

Range:

$|x| \leqslant 227{\cdot}95$

▼	A	00
e^x	4	01
+	E	02
(	6	03
÷	G	04
)	6	05
÷	G	06
+	E	07
=	–	08
stop	0	09
▼	A	10
goto	2	11
0	0	12
0	0	13
		14
		15
		16
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HYPERBOLIC FUNCTIONS

Cosech x

Execution:

x / RUN / cosech x

Range:

$1{\cdot}0017 \times 10^{-4} \leqslant |x| \leqslant 227{\cdot}95$

($|x| > 227{\cdot}95$ may give wrong result without E)

▼	A	00
e^x	4	01
—	F	02
(	6	03
÷	G	04
)	6	05
÷	G	06
+	E	07
=	—	08
stop	0	09
▼	A	10
goto	2	11
0	0	12
0	0	13
		14
		15
		16
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HYPERBOLIC FUNCTIONS

Coth x

Execution:

x / RUN / coth x

Range:

$1{\cdot}0016 \times 10^{-4} \leqslant |x| \leqslant 113{\cdot}97$

(out-of-range may give wrong result without E)

+	E	00
=	–	01
▼	A	02
e^x	4	03
–	F	04
#	3	05
1	1	06
÷	G	07
+	E	08
+	E	09
#	3	10
1	1	11
=	–	12
stop	0	13
▼	A	14
goto	2	15
0	0	16
0	0	17
		18
		19
		20
		21
		22
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		31
		32
		33
		34
		35

HYPERBOLIC FUNCTIONS

All the hyperbolic functions

▼	A	00
e^x	4	01
+	E	02
#	3	03
1	1	04
÷	G	05
+	E	06
–	F	07
#	3	08
1	1	09
–	F	10
=	–	11
▼	A	12
arctan	9	13
+	E	14
=	–	15
sto	2	16
tan	9	17
stop	0	18
÷	G	19
=	–	20
stop	0	21
rcl	5	22
cos	8	23
÷	G	24
=	–	25
stop	0	26
÷	G	27
=	–	28
stop	0	29
rcl	5	30
sin	7	31
stop	0	32
÷	G	33
=	–	34
stop	0	35

Execution:

x / RUN / sinh x / RUN / cosech x / RUN / cosh x / RUN / sech x / RUN / tanh x / RUN / coth x /

Range:

$1{\cdot}0017 \times 10^{-4} \leqslant |x| \leqslant 7{\cdot}8566$

HYPERBOLIC FUNCTIONS

The gudermannian program

Enables all the hyperbolic functions to be calculated with suitable execution sequences.

Formulae:

$$\mathrm{gd}\,x = 2 \arctan \tanh \frac{x}{2}$$

$$\sinh x = \tan \mathrm{gd}\,x$$

$$\mathrm{cosech}\,x = \cot \mathrm{gd}\,x$$

$$\cosh x = \sec \mathrm{gd}\,x$$

$$\mathrm{sech}\,x = \cos \mathrm{gd}\,x$$

$$\tanh x = \sin \mathrm{gd}\,x$$

$$\coth x = \mathrm{cosec}\,\mathrm{gd}\,x$$

Execution:

x / RUN / gd x / ▲▼ / sin / tanh x / ÷ / = / coth x

x / RUN / gd x / ▲▼ / cos / sech x / ÷ / = / cosh x

x / RUN / gd x / ▲▼ / tan / sinh x / ÷ / = / cosech x

This program can be used inside parentheses and does not affect memory.

Accuracy is less than that of individual hyperbolic function programs.

Range:

$|x| \leqslant 227{\cdot}95$ for gd x

$|x| \leqslant 7{\cdot}8566$ for hyperbolic functions

▼	A	00
e^x	4	01
+	E	02
#	3	03
1	1	04
÷	G	05
+	E	06
–	F	07
#	3	08
1	1	09
–	F	10
=	–	11
▼	A	12
arctan	9	13
+	E	14
=	–	15
stop	0	16
▼	A	17
goto	2	18
0	0	19
0	0	20
		21
		22
		23
		24
		25
		26
		27
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		31
		32
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		34
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INVERSE HYPERBOLIC FUNCTIONS

All the inverse hyperbolic functions can be obtained from the following program.

Execution:

▲▼ / ▲▼ / goto / 1 / 2 / x / RUN* / $\sinh^{-1} x$
or
x / RUN / $+\cosh^{-1} x$ / RUN / $-\cosh^{-1} x$
or
▲▼ / ▲▼ / goto / 2 / 0 / x / RUN / $\tanh^{-1} x$
or
▲▼ / ▲▼ / goto / 1 / 3 / x / RUN* / $\mathrm{cosech}^{-1} x$
or
▲▼ / ▲▼ / goto / 0 / 1 / x / RUN / $+\mathrm{sech}^{-1} x$ / RUN / $-\mathrm{sech}^{-1} x$
or
▲▼ / ▲▼ / goto / 1 / 9 / x / RUN / $\coth^{-1} x$

* For *negative* x press / RUN / a second time when evaluating $\sinh^{-1} x$ and $\mathrm{cosech}^{-1} x$ to get the correct answer.

Range:

$\sinh^{-1} x$	$10^{-49} \leqslant \lvert x\rvert \leqslant 577{\cdot}35$	
$\cosh^{-1} x$	$1 \leqslant x \leqslant 3162{\cdot}2$	No E if x –ve
$\tanh^{-1} x$	$-0{\cdot}99999 \leqslant x \leqslant 0{\cdot}99999$	
$\mathrm{cosech}^{-1} x$	$0{\cdot}001732 \leqslant \lvert x\rvert \leqslant 10^{49}$	
$\mathrm{sech}^{-1} x$	$3{\cdot}162278 \times 10^{-4} \leqslant x \leqslant 1$	No E if x –ve
$\coth^{-1} x$	$1{\cdot}0001 \leqslant \lvert x\rvert \leqslant 10^{99}$	

÷	G	00
X	·	01
–	F	02
+	E	03
#	3	04
1	1	05
=	–	06
√x	1	07
▼	A	08
goto	2	09
2	2	10
0	0	11
÷	G	12
X	·	13
+	E	14
#	3	15
1	1	16
=	–	17
√x	1	18
÷	G	19
–	F	20
+	E	21
#	3	22
1	1	23
÷	G	24
+	E	25
–	F	26
#	3	27
1	1	28
=	–	29
√x	1	30
ln	4	31
stop	0	32
–	F	33
=	–	34
stop	0	35

MODULO ARITHMETIC
('Clock Arithmetic')

Base 7 is used as an example.

The program completes a calculation and works out the remainder when the result is divided by 7. Neither the brackets nor the memory are used, so that the operation of / RUN / is exactly that of / = / .

For other bases, insert the base at steps 03, 11 and 14. Change the address at steps 18 and 19 to 14 if a two digit base is used, 16 for a three digit base, etc.

Execution may take a long time if very large numbers are involved.

Example:

/ 3 / X / 5 / RUN / 1 / + / 8 / RUN / 2 / etc.

–	F	00
+	E	01
#	3	02
7	7	03
–	F	04
▼	A	05
gin	1	06
0	0	07
0	0	08
–	F	09
#	3	10
7	7	11
+	E	12
#	3	13
7	7	14
=	–	15
▼	A	16
gin	1	17
1	1	18
2	2	19
stop	0	20
▼	A	21
goto	2	22
0	0	23
0	0	24
		25
		26
		27
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		29
		30
		31
		32
		33
		34
		35

PRIME FACTORISATION

To find the prime factors of a number N.

Pre-execution:

2 / ▲▼ / sto / ▲▼ / ▲▼ / goto / 0 / 0 / C/CE /

Execution:

N / RUN / a_1 / RUN / N_1 / RUN / a_2 / RUN / N_2 / · · · / a_r / RUN / 1

where
$a_1, a_2, a_3, \cdots a_r$ are the prime factors of N
and
$N_1, N_2, \cdots$ are the residues defined by

$$N_1 = \frac{N}{a_1}, \quad N_2 = \frac{N}{a_1 a_2}, \quad N_3 = \frac{N}{a_1 a_2 a_3}, \text{ etc.}$$

Pressing / RUN / after 1 has been displayed will cause the machine to go into an infinite loop.

Warning: Long execution times are possible for large values of N or for numbers with large prime factors.

÷.	G	00
(	6	01
–	F	02
+	E	03
rcl	5	04
–	F	05
▼	A	06
gin	1	07
0	0	08
2	2	09
=	–	10
▼	A	11
gin	1	12
2	2	13
4	4	14
rcl	5	15
stop	0	16
)	6	17
=	–	18
stop	0	19
▼	A	20
goto	2	21
0	0	22
0	0	23
rcl	5	24
+	E	25
#	3	26
1	1	27
=	–	28
sto	2	29
#	3	30
1	1	31
=	–	32
)	6	33
=	–	34
=	–	35

PRIME NUMBER TESTING

To find whether a number n is prime, choose any integer $m \geqslant \sqrt{n}$.

Then use the execution sequence:

n / RUN / m / RUN /

The result will be the largest number less than or equal to m which divides n. If the result is 1 then n is prime.

To test another number, pre-execute with:

/ ▲▼ / ▲▼ / goto / 0 / 0 /

Note: Long execution times are possible for large numbers.

sto	2	00
stop	0	01
▼	A	02
MEx	5	03
+	E	04
(	6	05
–	F	06
+	E	07
rcl	5	08
–	F	09
▼	A	10
gin	1	11
0	0	12
6	6	13
=	–	14
▼	A	15
gin	1	16
2	2	17
1	1	18
rcl	5	19
stop	0	20
rcl	5	21
–	F	22
#	3	23
1	1	24
=	–	25
sto	2	26
#	3	27
0	0	28
=	–	29
)	6	30
▼	A	31
goto	2	32
0	0	33
4	4	34
		35

FACTORIALS

sto	2	00
–	F	01
#	3	02
2	2	03
+	E	04
▼	A	05
gin	1	06
2	2	07
1	1	08
#	3	09
1	1	10
X	·	11
▼	A	12
MEx	5	13
=	–	14
▼	A	15
MEx	5	16
▼	A	17
goto	2	18
0	0	19
1	1	20
=	–	21
rcl	5	22
stop	0	23
▼	A	24
goto	2	25
0	0	26
0	0	27
		28
		29
		30
		31
		32
		33
		34
		35

Execution:

n / RUN / n!

Restriction:

$1 \leqslant n \leqslant 69$

Note: The program may be used within brackets. It does, however, use the memory. Thus, to calculate

$$\frac{15!}{6!\ 10!}$$

a possible execution sequence is:

15 / RUN / ÷ / ▲▼ / (/ 10 / RUN / ▲▼ /) / ÷ / ▲▼ / (/ 6 / RUN / ▲▼ /) / = / answer

FACTORIALS OF LARGE NUMBERS

This program calculates ln (n!) for n greater than about 25.

Reasonably accurate results are given for n greater than 10.

(The program uses Stirling's approximation, $n! \doteq \sqrt{2\pi n}\, e^{-n} n^{n}$)

Execution:

n / RUN / ln (n!)

sto	2	00
+	E	01
#	3	02
·	A	03
5	5	04
X	·	05
(	6	06
rcl	5	07
ln	4	08
)	6	09
—	F	10
rcl	5	11
+	E	12
#	3	13
·	A	14
9	9	15
1	1	16
8	8	17
9	9	18
=	—	19
stop	0	20
▼	A	21
goto	2	22
0	0	23
0	0	24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

THE GAMMA AND PI FUNCTIONS

$\Gamma(n+1) = \pi(n) = n!$	when n is an integer
$\Gamma(x+1) = x\,\Gamma(x)$	for $x > 0$
$\Gamma(0)$ is undefined	$\pi(0) = \Gamma(1) = 1$
$\Gamma(\tfrac{1}{2}) = \sqrt{\pi}$	$\pi(\tfrac{1}{2}) = \tfrac{1}{2}\sqrt{\pi}$
$\Gamma(1) = 1$	$\pi(1) = \Gamma(2) = 1$

By interpolation:

$$\Gamma(n+\delta) \bumpeq (n + \tfrac{1}{2}\delta - \tfrac{1}{2})^{\delta}\,\Gamma(n) \qquad 0 \leqslant \delta \leqslant 1$$

$$\therefore \pi(\delta) \bumpeq (n + \tfrac{1}{2}\delta - \tfrac{1}{2})^{\delta} \prod_{r=1}^{n-1} \frac{(r)}{r+\delta} \qquad 0 \leqslant \delta \leqslant 1$$

$$\Gamma(\delta) = \frac{\pi(\delta)}{\delta} \bumpeq \frac{(n + \tfrac{1}{2}\delta - \tfrac{1}{2})^{\delta}}{\delta} \prod_{r=1}^{n} \frac{r}{(r+\delta)}$$

$0 \leqslant \delta \leqslant 1$

n should be suitably large for the accuracy required.

n = 20 gives high accuracy

n = 5 gives reasonable accuracy for most purposes

e.g. $\pi(\tfrac{1}{2}) = \tfrac{1}{2}\sqrt{\pi} = 0{\cdot}8862269$

n = 5 gives $\pi(\tfrac{1}{2}) \bumpeq 0{\cdot}885547$

n = 20 gives $\pi(\tfrac{1}{2}) \bumpeq 0{\cdot}8861174$

Execution:

▲▼ / ▲▼ / goto / 0 / 0 /

n / RUN / δ / RUN / n – 1 / RUN / n – 2 / RUN / · · · / 2 / RUN / 1 / RUN / $\pi(\delta)$ / ▲▼ / ▲▼ / goto / 3 / 2 / RUN / $\Gamma(\delta)$

+	E	00
+	E	01
stop	0	02
sto	2	03
–	F	04
#	3	05
1	1	06
÷	G	07
#	3	08
2	2	09
=	–	10
ln	4	11
X	·	12
rcl	5	13
=	–	14
▼	A	15
e^x	4	16
÷	G	17
(	6	18
stop	0	19
÷	G	20
rcl	5	21
÷	G	22
+	E	23
#	3	24
1	1	25
=	–	26
)	6	27
▼	A	28
goto	2	29
1	1	30
7	7	31
rcl	5	32
)	6	33
=	–	34
stop	0	35

FIBONACCI NUMBERS

sto	2	00
#	3	01
1	1	02
+	E	03
stop	0	04
▼	A	05
MEx	5	06
▼	A	07
goto	2	08
0	0	09
3	3	10
		11
		12
		13
		14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

Each number in the sequence is the sum of the previous two.

Execution:

C/CE / RUN / F_1 / RUN / F_2 / RUN / · · ·

NUMBER BASE CONVERSIONS

Decimal to binary (fractions)

Given a decimal x, $0 \leqslant x \leqslant 1$, this program calculates the binary expansion of x to any number of places.

Suppose $x = 0{\cdot}d_1 d_2 \ldots$ (binary)

Execution:

x / RUN / d_1 / RUN / d_2 / RUN / d_3 / . . .

To calculate the expansion of another decimal y, press

/C/CE / C/CE / ▲▼ / ▲▼ / goto / 0 / 0 / y / RUN / . . .

etc.

Notes:

1. To convert decimal integers to binary use the program on page 31.
2. No program for converting decimal fractions to bases other than 2 is provided.

sto	2	00
#	3	01
1	1	02
=	–	03
▼	A	04
MEx	5	05
–	F	06
(	6	07
rcl	5	08
÷	G	09
#	3	10
2	2	11
=	–	12
sto	2	13
)	6	14
–	F	15
(	6	16
▼	A	17
gin	1	18
2	2	19
4	4	20
#	3	21
1	1	22
+	E	23
#	3	24
0	0	25
X	·	26
stop	0	27
rcl	5	28
)	6	29
+	E	30
rcl	5	31
▼	A	32
goto	2	33
0	0	34
6	6	35

NUMBER BASE CONVERSIONS

Decimal integer to base m

This program expresses any integer in any base.

Suppose $x = a_1 \cdots a_r$ in base m.

Execution:

m / RUN / x / RUN / a_r / RUN / a_{r-1} / · · · / a_1 / RUN / m

Note that the digits are produced in reverse order and that the machine tells you that all the digits have been shown by displaying the base m.

The sequence can be repeated for a new x and/or m. If the same m is required there is no need to re-enter it because it is already in the display.

Note: To convert decimal fractions to base 2, use the program on page 30.

sto	2	00
stop	0	01
–	F	02
(	6	03
+	E	04
#	3	05
1	1	06
–	F	07
+	E	08
rcl	5	09
–	F	10
▼	A	11
gin	1	12
0	0	13
7	7	14
+	E	15
rcl	5	16
–	F	17
#	3	18
1	1	19
=	–	20
)	6	21
stop	0	22
÷	G	23
rcl	5	24
–	F	25
–	F	26
▼	A	27
gin	1	28
0	0	29
2	2	30
=	–	31
rcl	5	32
stop	0	33
=	–	34
=	–	35

NUMBER BASE CONVERSIONS

Binary to decimal (integers)

Binary is $a_n \cdots a_o$

Execution:

a_n / RUN / a_{n-1} / RUN / $\cdots$ / a_o / = / answer

+	E	00
+	E	01
stop	0	02
▼	A	03
goto	2	04
0	0	05
0	0	06
		07
		08
		09
		10
		11
		12
		13
		14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

NUMBER BASE CONVERSIONS

Binary fraction to decimal

#	3	00
1	1	01
=	–	02
sto	2	03
(	6	04
stop	0	05
▼	A	06
MEx	5	07
÷	G	08
#	3	09
2	2	10
×	·	11
▼	A	12
MEx	5	13
)	6	14
+	E	15
▼	A	16
goto	2	17
0	0	18
4	4	19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

If number is:

$0 \cdot b_1 b_2 \cdots b_k$

Execution:

RUN / b_1 / RUN / b_2 / $\cdots$ / b_k / RUN / answer

At each stage the answer so far is displayed.

Fraction base m to decimal

Exactly the same except / 2 / at step 10 is replaced by the appropriate base.

NUMBER BASE CONVERSIONS

Binary to decimal (integers, fractions or mixed numbers)

Binary is $a_n \ldots a_o \cdot b_1 \ldots b_m$

Execution:

C/CE / RUN / a_n / RUN / a_{n-1} / RUN / · · · / RUN / a_o / – / RUN / b_1 / RUN / b_2 / · · · / b_m / RUN / answer

Notes:

1. The / – / corresponding to the 'decimal' point must be entered even if the number is an integer.
2. The correct answer will be given if:

   $a_n = 1$

   $n \geqslant 1$

   $a_o = 1$ or 0

   or just $\cdot b_1 \cdots$ (· entered as –)

To re-use:

C/CE / C/CE / ▲▼ / ▲▼ / goto / 0 / 0

+	E	00
+	E	01
stop	0	02
–	F	03
–	F	04
▼	A	05
gin	1	06
0	0	07
0	0	08
sto	2	09
=	–	10
#	3	11
1	1	12
=	–	13
▼	A	14
MEx	5	15
+	E	16
(	6	17
stop	0	18
▼	A	19
MEx	5	20
÷	G	21
#	3	22
2	2	23
X	·	24
▼	A	25
MEx	5	26
)	6	27
▼	A	28
goto	2	29
1	1	30
6	6	31
		32
		33
		34
		35

NUMBER BASE CONVERSIONS

Base m to decimal (integers)

sto	2	00
stop	0	01
X	·	02
rcl	5	03
+	E	04
▼	A	05
goto	2	06
0	0	07
1	1	08
		09
		10
		11
		12
		13
		14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

Number is $a_n \; a_{n-1} \cdots a_o$

Execution:

m / RUN / a_n / RUN / a_{n-1} / RUN / · · · / a_o / = / answer

To re-use with same m:

C/CE / RUN / a'_n · · ·

NUMBER BASE CONVERSIONS

Base m to decimal (integers, fractions or mixed numbers)

Number is: $a_n \ldots a_o \cdot b_1 \ldots b_p$

Example:

m = 7

Execution:

C/CE / RUN / a_n / RUN / a_{n-1} / · · · / RUN / a_o / – / RUN / b_1 / RUN / b_2 / · · · / RUN / b_p / RUN / answer

Notes:

1. Insert value of m at 02 and 25.
2. If two digit base is used, insert at 02, 03, move the next 22 steps down one, insert the base again at 26, 27, and substitute / 1 / 9 / for / 1 / 8 / in the last two steps.

X	·	00
#	3	01
7	7	02
+	E	03
stop	0	04
–	F	05
–	F	06
▼	A	07
gin	1	08
0	0	09
0	0	10
sto	2	11
=	–	12
#	3	13
1	1	14
=	–	15
▼	A	16
MEx	5	17
+	E	18
(	6	19
stop	0	20
▼	A	21
MEx	5	22
÷	G	23
#	3	24
7	7	25
X	·	26
▼	A	27
MEx	5	28
)	6	29
▼	A	30
goto	2	31
1	1	32
8	8	33
		34
		35

SERIES

Natural numbers

$$(1 + 2 + \cdots + n) = \frac{1}{2}n(n + 1)$$

Execution:

n / RUN / [illegible]

+	E	00
(	6	01
X	·	02
)	6	03
÷	G	04
#	3	05
2	2	06
=	–	07
stop	0	08
▼	A	09
goto	2	10
0	0	11
0	0	12
		13
		14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

SERIES

Squares of natural numbers

sto	2	00
+	E	01
+	E	02
#	3	03
3	3	04
X	·	05
rcl	5	06
+	E	07
#	3	08
1	1	09
X	·	10
rcl	5	11
÷	G	12
#	3	13
6	6	14
=	—	15
stop	0	16
▼	A	17
goto	2	18
0	0	19
0	0	20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

$$(1 + 4 + 9 + \cdots + n^2) = \frac{1}{6}n(n + 1)(2n + 1)$$

Execution:

n / RUN / [illegible]

SERIES

Cubes of natural numbers

$$(1 + 8 + 27 + \cdots n^3) = \frac{1}{4} n^2 (n + 1)^2$$

Execution:

n / RUN / sum

+	E	00
(	6	01
X	·	02
)	6	03
X	·	04
÷	G	05
#	3	06
4	4	07
=	—	08
stop	0	09
▼	A	10
goto	2	11
0	0	12
0	0	13
		14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

ARITHMETIC SERIES

First term = a
Common difference = d
N terms

$$\text{sum} = N\left(a + \frac{(N-1)d}{2}\right)$$

Execution:

a / RUN / N / RUN / d / RUN / sum

+	E	00
(	6	01
stop	0	02
sto	2	03
–	F	04
#	3	05
1	1	06
÷	G	07
#	3	08
2	2	09
×	·	10
stop	0	11
)	6	12
×	·	13
rcl	5	14
=	–	15
stop	0	16
▼	A	17
goto	2	18
0	0	19
0	0	20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

ARITHMETIC SERIES

+	E	00
stop	0	01
÷	G	02
#	3	03
2	2	04
X .	·	05
stop	0	06
=	—	07
stop	0	08
▼	A	09
goto	2	10
0	0	11
0	0	12
		13
		14
		15
		16
		17
		18
		19
		20
		21
		22
		23
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		31
		32
		33
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First term = a

Last term = l

N terms

$$\text{sum} = \frac{N(a + l)}{2}$$

Execution:

a / RUN / l / RUN / N / RUN / sum

GEOMETRIC SERIES

$$S = a + ar + \cdots + ar^{N-1} = \frac{a(1-r^n)}{(1-r)}$$

First term = a

Common ratio = r

N terms

Restrictions:

$r > 0, \quad r \neq 1$

Execution:

a / RUN / r / RUN / N / RUN / sum

÷	G	00
(	6	01
stop	0	02
sto	2	03
—	F	04
#	3	05
1	1	06
=	—	07
)	6	08
X	·	09
(	6	10
rcl	5	11
ln	4	12
X	·	13
stop	0	14
=	—	15
▼	A	16
e^x	4	17
—	F	18
#	3	19
1	1	20
=	—	21
)	6	22
=	—	23
stop	0	24
▼	A	25
goto	2	26
0	0	27
0	0	28
		29
		30
		31
		32
		33
		34
		35

INFINITE GEOMETRIC SERIES

÷	G	00
(	6	01
#	3	02
1	1	03
–	F	04
stop	0	05
)	6	06
=	–	07
stop	0	08
▼	A	09
goto	2	10
0	0	11
0	0	12
		13
		14
		15
		16
		17
		18
		19
		20
		21
		22
		23
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		31
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		33
		34
		35

$$S = a + ar + ar^2 + \cdots = \frac{a}{1 - r}$$

Restriction:

$|r| < 1$

Execution:

a / RUN / r / RUN / sim

ARITHMETIC – GEOMETRIC SERIES (infinite)

$$S = a + (a+d)r + (a+2d)r^2 + \cdots + (a+nd)r^n + \cdots$$

$$= \frac{a + \dfrac{dr}{1-r}}{1-r}$$

Restriction:

$|r| < 1$

Execution:

r / RUN / d / RUN / a / RUN / sum

–	F	00
#	3	01
1	1	02
–	F	03
÷	G	04
×	·	05
(	6	06
–	F	07
#	3	08
1	1	09
×	·	10
stop	0	11
+	E	12
stop	0	13
)	6	14
=	–	15
stop	0	16
▼	A	17
goto	2	18
0	0	19
0	0	20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

SUMMING SERIES IN GENERAL

$\sum_{1}^{N} a(n)$, some function a.

Examples:

1. $1 + 4 + 9 + \cdots + N^2$ $\qquad a(n) = n^2$

2. $\left(1 + \frac{1}{1}\right) + \left(8 + \frac{1}{4}\right) + \cdots + \left(N^3 + \frac{1}{N^2}\right)$

   $a(n) = n^3 + \frac{1}{n^2}$ etc.

Write a program segment which evaluates a(n) when n is in memory; parentheses may not be used. The segment may be up to 15 steps long, any final / = / stop / being omitted. Fill up any unused steps with / = / · · · / = /.

Examples for above:

1. n^2 $\qquad$ rcl / X /

2. $n^3 + \frac{1}{n^2}$ $\qquad$ write as $(n^5 + 1) \div n^2$

   rcl / X / X / X / rcl / + / # / 1 / ÷ / rcl / rcl / = / = / = /

   Then use the program as shown.

Pre-execution:

Clear memory with C/CE / ▲▼ / sto /

Execution:

N / RUN / a(1) + a(2) + · · · + a(n)

=	–	00
▼	A	01
MEx	5	02
+	E	03
(	6	04
		05
Y		06
O		07
U		08
R		09
		10
S		11
E		12
G		13
M		14
E		15
N		16
T		17
		18
		19
)	6	20
=	–	21
▼	A	22
MEx	5	23
–	F	24
#	3	25
1	1	26
–	F	27
–	F	28
▼	A	29
gin	1	30
0	0	31
0	0	32
=	–	33
rcl	5	34
stop	0	35

SERIES

$a(x_1) + a(x_2) + \cdots + a(x_n)$

Write a program segment to evaluate $a(x_i)$ without using parentheses; the memory may be used.

Then use the following program:

/ (/ stop / · · · seg · · · /) / + / ▼ / goto / 0 / 0 /

Execution:

RUN / x_1 / RUN / x_2 / · · · / x_n / RUN / sum

At each step the sum so far is displayed.

Example:

To find $\Sigma \tan\left(x^2 + \frac{1}{x}\right)$

Express $x^2 + \frac{1}{x}$ as $\frac{x^3 + 1}{x}$

Program segment is then:

/ sto / X / X / rcl / + / # / 1 / ÷ / rcl / = / tan /

and so program is as shown.

The segment may be up to 32 steps long, by omitting / ▼ / goto / 0 / 0 / at the end and filling any empty steps with / = /.

(	6	00
stop	0	01
sto	2	02
X	·	03
X	·	04
rcl	5	05
+	E	06
#	3	07
1	1	08
÷	G	09
rcl	5	10
=	–	11
tan	9	12
)	6	13
+	E	14
▼	A	15
goto	2	16
0	0	17
0	0	18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

HARMONIC ADDITION

÷	G	00
+	E	01
(	6	02
÷	G	03
=	–	04
stop	0	05
÷	G	06
)	6	07
▼	A	08
goto	2	09
0	0	10
1	1	11
		12
		13
		14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
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Resistors in parallel, capacitors in series, lenses in series, etc.

$$\frac{1}{x} = \frac{1}{x_1} + \cdots + \frac{1}{x_n}$$

Execution:

x_1 / RUN / x_2 / RUN / · · · / x_n / RUN / x

At each step the harmonic sum so far is displayed.

PYTHAGOREAN ADDITION

Geometry, electricity

$$x = \sqrt{x_1^2 + \cdots + x_n^2}$$

Execution:

x_1 / RUN / x_2 / RUN / $\cdots$ / x_n / RUN / x

At each step the intermediate result $\sqrt{x_1^2 + \cdots + x_i^2}$ is displayed.

×	·	00
+	E	01
(	6	02
$\sqrt{x}$	1	03
stop	0	04
×	·	05
)	6	06
▼	A	07
goto	2	08
0	0	09
1	1	10
		11
		12
		13
		14
		15
		16
		17
		18
		19
		20
		21
		22
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		30
		31
		32
		33
		34
		35

ARITHMETIC MEAN

X	·	00
(	6	01
#	3	02
1	1	03
=	–	04
sto	2	05
)	6	06
+	E	07
(	6	08
stop	0	09
÷	G	10
rcl	5	11
)	6	12
÷	G	13
(	6	14
#	3	15
1	1	16
+	E	17
rcl	5	18
÷	G	19
▼	A	20
MEx	5	21
)	6	22
▼	A	23
goto	2	24
0	0	25
7	7	26
		27
		28
		29
		30
		31
		32
		33
		34
		35

Pre-execution:

C/CE / C/CE / ▲▼ / ▲▼ / goto / 0 / 0

Execution:

x_1 / RUN / x_2 / RUN / · · · / x_n / RUN / arithmetic mean

At each stage the arithmetic mean so far is displayed.

GEOMETRIC MEAN

ln	4	00
X	·	01
(	6	02
#	3	03
1	1	04
=	–	05
sto	2	06
)	6	07
+	E	08
(	6	09
▼	A	10
e^x	4	11
stop	0	12
ln	4	13
÷	G	14
rcl	5	15
)	6	16
÷	G	17
(	6	18
#	3	19
1	1	20
+	E	21
rcl	5	22
÷	G	23
▼	A	24
MEx	5	25
)	6	26
▼	A	27
goto	2	28
0	0	29
8	8	30
		31
		32
		33
		34
		35

Pre-execution:

C/CE / C/CE / ▲▼ / ▲▼ / goto / 0 / 0

Execution:

x_1 / RUN / x_2 / RUN / · · · / x_n / RUN / geometric mean

At each stage the geometric mean so far is displayed.

HARMONIC MEAN

$$\frac{1}{H} = \frac{1}{n}\left(\frac{1}{x_1} + \cdots + \frac{1}{x_n}\right)$$

Pre-execution:

C/CE / C/CE / ▲▼ / ▲▼ / goto / 0 / 0

Execution:

x_1 / RUN / x_2 / RUN / · · · / x_n / RUN / harmonic mean

At each stage the harmonic mean so far is displayed.

÷	G	00
X	·	01
(	6	02
#	3	03
1	1	04
=	—	05
sto	2	06
)	6	07
+	E	08
(	6	09
÷	G	10
=	—	11
stop	0	12
÷	G	13
÷	G	14
rcl	5	15
)	6	16
÷	G	17
(	6	18
#	3	19
1	1	20
+	E	21
rcl	5	22
÷	G	23
▼	A	24
MEx	5	25
)	6	26
▼	A	27
goto	2	28
0	0	29
8	8	30
		31
		32
		33
		34
		35

ROOT MEAN SQUARE

$$R = \sqrt{\frac{(x_1^2 + \cdots + x_n^2)}{n}}$$

Pre-execution:

C/CE / C/CE / ▲▼ / ▲▼ / goto / 0 / 0

Execution:

x_1 / RUN / x_2 / · · · / x_n / RUN / root-mean-square

At each stage the r.m.s. so far is displayed.

×	·	00
×	·	01
(	6	02
#	3	03
1	1	04
=	–	05
sto	2	06
)	6	07
+	E	08
(	6	09
$\sqrt{x}$	1	10
stop	0	11
×	·	12
÷	G	13
rcl	5	14
)	6	15
÷	G	16
(	6	17
#	3	18
1	1	19
+	E	20
rcl	5	21
÷	G	22
▼	A	23
MEx	5	24
)	6	25
▼	A	26
goto	2	27
0	0	28
8	8	29
		30
		31
		32
		33
		34
		35

QUADRATIC EQUATIONS

$ax^2 + bx + c = 0$

Roots x_1, x_2 if real

$R \pm iI$ if complex

Execution:

a / RUN / b / RUN / c / RUN /

- x_1 / RUN / x_2 / RUN / RUN / C/CE / C/CE / if roots are real
- I* / C/CE / RUN / R / if roots are complex

* error symbol displayed

After the sequence a / RUN / b / RUN / c / RUN / the display shows *either* (if the roots are real) the larger real root with no error indication *or* (if the roots are complex) the imaginary part and the error symbol. Continue with the appropriate execution sequence.

The error symbol will tell you whether the roots are complex. The sequence / RUN / RUN / C/CE / shown above after (x_2) is necessary before entering a new equation to be solved.

+	E	00
÷	G	01
–	F	02
X	·	03
sto	2	04
stop	0	05
=	–	06
▼	A	07
MEx	5	08
X	·	09
stop	0	10
+	E	11
+	E	12
(	6	13
rcl	5	14
X	·	15
)	6	16
+	E	17
▼	A	18
gin	1	19
3	3	20
2	2	21
$\sqrt{x}$	1	22
▼	A	23
MEx	5	24
–	F	25
stop	0	26
rcl	5	27
–	F	28
rcl	5	29
=	–	30
stop	0	31
$\sqrt{x}$	1	32
stop	0	33
rcl	5	34
stop	0	35

CUBIC EQUATIONS
by an iterative method

$ax^3 + bx^2 + cx + d = 0$

Formula:

$$x_{k+1} = \frac{2ax_k^3 + bx_k^2 - d}{3ax_k^2 + 2bx_k + c}$$ (based on Newton-Raphson method)

(Fill in your own values of 2a, b, d, etc.; if any of these are negative change the + or − preceding them to − or +)

Execution:

Choose any starting value x_o, say $-\frac{d}{c}$

x_o / RUN / x_1 / RUN / x_2 / · · ·

If the sequence converges, the limit will solve the equation.

If the sequence does not converge, try a new starting value.

The sequence will usually converge to the root closest to the starting value and so by trying different starting values all the roots should be obtained.

* where $a_1 a_2$ is the two digit number 3a; if $3a < 10$ then enter $a_1 = 0$ and a_2 as the value of 3a. Similarly $b_1 b_2$ is 2b.

	sto	2	00
	X	·	01
	#	3	02
	a	a	03
	+	E	04
	+	E	05
	#	3	06
	b	b	07
	X	·	08
	rcl	5	09
	X	·	10
	rcl	5	11
	—	F	12
	#	3	13
	d	d	14
	÷	G	15
	(	6	16
	#	3	17
*	a_1	a_1	18
	a_2	a_2	19
	X	·	20
	rcl	5	21
	+	E	22
	#	3	23
*	b_1	b_1	24
	b_2	b_2	25
	X	·	26
	rcl	5	27
	+	E	28
	#	3	29
	c	c	30
	=	—	31
	)	6	32
	=	—	33
	stop	0	34
	=	—	35

POLYNOMIALS

sto	2	00
stop	0	01
X	·	02
rcl	5	03
+	E	04
▼	A	05
goto	2	06
0	0	07
1	1	08
		09
		10
		11
		12
		13
		14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

To evaluate

$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_o = p(x)$

Execution:

x / RUN / a_n / RUN / a_{n-1} / · · · / a_1 / RUN / a_o / = / result

To use again: (with different x)

▲▼ / ▲▼ / goto / 0 / 0 / before execution

Notes:

1. The individual results after each / RUN / are the coefficients of the polynomial q(x) where q(t) = p(t) / (t − x).
2. If p(x) = 0, x is a root and q(x) is the quotient polynomial which can be solved for other roots of p(x).

POLYNOMIALS

To write a program to evaluate the same polynomial repeatedly

Example:

$p(x) = 5x^4 + 8x^3 - 3x^2 + 4{\cdot}2x + 1$

Method:

Express as $[\![\,[(5x+8)x-3]x+4{\cdot}2\,]\!]x+1$

Execution:

x / RUN / p(x) / y / RUN / p(y) · · · etc.

Note: If a coefficient is zero omit it together with the – or + sign preceding it. If the leading coefficient is 1, it may be omitted together with the multiplication sign which precedes it. See over for example.

sto	2	00
X	·	01
#	3	02
5	5	03
+	E	04
#	3	05
8	8	06
X	·	07
rcl	5	08
–	F	09
#	3	10
3	3	11
X	·	12
rcl	5	13
+	E	14
#	3	15
4	4	16
·	A	17
2	2	18
X	·	19
rcl	5	20
+	E	21
#	3	22
1	1	23
=	–	24
stop	0	25
▼	A	26
goto	2	27
0	0	28
0	0	29
		30
		31
		32
		33
		34
		35

POLYNOMIALS

first coefficient = 1, so omitted.

coefficient of x = 0, so omitted.

sto	2	00
+	E	01
#	3	02
2	2	03
X	·	04
rcl	5	05
X	·	06
rcl	5	07
+	E	08
#	3	09
3	3	10
=	–	11
stop	0	12
▼	A	13
goto	2	14
0	0	15
0	0	16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

Example:

To calculate $x^3 + 2x^2 + 3$

DIVISION OF A POLYNOMIAL BY A QUADRATIC

Division of the polynomial

$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_o$

by the quadratic divisor

$d(x) = x^2 + mx + n$

gives the quotient polynomial

$q(x) = b_{n-2} x^{n-2} + b_{n-3} x^{n-3} + \cdots + b_1 x + b_o$

with remainder

$r(x) = c_1 x + c_o$

Pre-execution:

▲▼ / ▲▼ / goto / 0 / 0 / C/CE / ▲▼ / sto /

Execution:

RUN / n / RUN / m / RUN / a_n / RUN / b_{n-2}
RUN / n / RUN / m / RUN / a_{n-1} / RUN / b_{n-3}
RUN / n / RUN / m / RUN / a_2 / RUN / b_o
RUN / n / RUN / m / RUN / a_1 / RUN / c_1
RUN / n / RUN / m / RUN / a_o / RUN / RUN /
m / RUN / 1 / RUN / c_o
/ RUN / RUN / completes execution

Results may be tabulated as below: e.g. to divide $x^6 - 4x^5 + 31x^4 - 96x^3 + 415x^2 - 652x + 1105$ by $x^2 + 2x + 3$:

r	n	m	a_r	b_{r-2}
6	3	2	1	1
5			−4	−6
4			31	40
3			−96	−158
2			415	611
1			−652	$-1400 = c_1$
0			1105	$-728 = c_o$

▼	A	00
MEx	5	01
X	·	02
stop	0	03
+	E	04
(	6	05
stop	0	06
X	·	07
rcl	5	08
)	6	09
−	F	10
stop	0	11
−	F	12
=	−	13
stop	0	14
▼	A	15
goto	2	16
0	0	17
0	0	18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

SOLVING A POLYNOMIAL

This is an iterative method to find a quadratic factor of a polynomial. When the polynomial has been reduced to quadratic factors, these can be solved to give the real or complex roots of the original polynomial.

Stage 1:

Choose a starting quadratic divisor

$d(x) = x^2 + mx + n$ (say)

Divide $p(x)$ by $d(x)$ to give a quotient $q(x)$ and remainder $r(x) = rx + s$

Stage 2:

Divide $q(x)$ again by $d(x)$ to give a new quotient $q'(x)$ and remainder $r'(x) = tx + u$

Stage 3:

Find the coefficients m' and n' of the next iterate of the quadratic divisor using this program

Execution:

u / RUN / t / RUN / m / RUN / n / RUN / D

p / RUN / t / RUN / u / RUN / s / RUN / r / RUN / t / RUN / – / + / n / = / n'

▲▼ / ▲▼ / goto / 2 / 5 / r / RUN / u / RUN / n / X / s / RUN / t / RUN / + / m / = / m'

$D = u^2 + nt^2 - mut$

$$m' = m + \frac{ru + nst}{D}$$

$$n' = n - \frac{rt + s(mt - u)}{D}$$

Re-enter the quadratic divisor program and iterate again with the new values of m' and n'. Repeat stages 1–3 until the values of m and n converge.

X	·	00
sto	2	01
–	F	02
(	6	03
rcl	5	04
X	·	05
stop	0	06
sto	2	07
X	·	08
stop	0	09
)	6	10
+	E	11
(	6	12
rcl	5	13
X	·	14
X	·	15
stop	0	16
)	6	17
=	–	18
sto	2	19
stop	0	20
X	·	21
stop	0	22
–	F	23
stop	0	24
X	·	25
stop	0	26
+	E	27
(	6	28
stop	0	29
X	·	30
stop	0	31
)	6	32
÷	G	33
rcl	5	34
stop	0	35

NUMERICAL INTEGRATION

Triangular interpolation

$$I = \frac{1}{2}h(y_o + 2y_1 + 2y_2 + \cdots + 2y_{n-1} + y_n)$$

Execution:

n / RUN / y_o / RUN / y_1 / RUN / y_2 / RUN / · · · / RUN / y_n / RUN / h / RUN /

—	F	00
#	3	01
1	1	02
=	—	03
sto	2	04
stop	0	05
+	E	06
(	6	07
rcl	5	08
—	F	09
#	3	10
1	1	11
=	—	12
sto	2	13
▼	A	14
gin	1	15
2	2	16
5	5	17
stop	0	18
+	E	19
)	6	20
▼	A	21
goto	2	22
0	0	23
6	6	24
stop	0	25
)	6	26
X	·	27
stop	0	28
÷	G	29
#	3	30
2	2	31
=	—	32
stop	0	33
=	—	34
=	—	35

NUMERICAL INTEGRATION

Simpson's Rule

$$I = \frac{1}{3}h(y_o + 4y_1 + 2y_2 + 4y_3 + \cdots + 4y_{n-1} + y_n)$$

(n must be even)

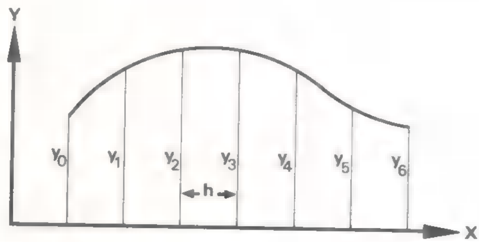
Execution:

n / – / 1 / = / RUN / y_1 / RUN / · · · / y_n / RUN / h / RUN / I

sto	2	00
stop	0	01
+	E	02
(	6	03
stop	0	04
+	E	05
+	E	06
)	6	07
+	E	08
(	6	09
rcl	5	10
–	F	11
#	3	12
2	2	13
=	–	14
sto	2	15
▼	A	16
gin	1	17
2	2	18
7	7	19
stop	0	20
+	E	21
)	6	22
▼	A	23
goto	2	24
0	0	25
2	2	26
stop	0	27
)	6	28
X	·	29
stop	0	30
÷	G	31
#	3	32
3	3	33
=	–	34
stop	0	35

NUMERICAL INTEGRATION

Weddle Formula



$$\text{Integral} = \frac{3h}{10}(y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6)$$

Execution:

y_0 / RUN / y_1 / RUN / y_2 / RUN / y_3 / RUN / y_4 / RUN / y_5 / RUN / y_6 / RUN / h_1 / = / integral

+	E	00
(	6	01
stop	0	02
X	·	03
#	3	04
5	5	05
=	–	06
)	6	07
+	E	08
stop	0	09
+	E	10
(	6	11
stop	0	12
X	·	13
#	3	14
6	6	15
=	–	16
)	6	17
+	E	18
stop	0	19
+	E	20
(	6	21
stop	0	22
X	·	23
#	3	24
5	5	25
=	–	26
)	6	27
+	E	28
stop	0	29
X	·	30
#	3	31
·	A	32
3	3	33
X	·	34
stop	0	35

COMPLEX NUMBERS

$z = x + iy$

To find magnitude and argument.

Execution:

If $y = 0$, then $z = |x|$ and arg z = (0 if $x \geqslant 0$, π if $x < 0$)

Otherwise, x / RUN / y / RUN / [illegible] / RUN / arg z

To find x and y given arg z and |z|

($-\pi \leqslant$ arg $z \leqslant \pi$)

If arg z is 0, then $x = |z|$ and $y = 0$

If arg z is π, then $x = -|z|$ and $y = 0$

Otherwise use polar-cartesian program, execution as follows:

|z| / RUN / arg z / RUN / [illegible] / RUN / y

÷	G	00
(	6	01
×	·	02
÷	G	03
stop	0	04
sto	2	05
+	E	06
rcl	5	07
×	·	08
rcl	5	09
=	–	10
$\sqrt{x}$	1	11
stop	0	12
)	6	13
+	E	14
#	3	15
1	1	16
÷	G	17
#	3	18
2	2	19
=	–	20
$\sqrt{x}$	1	21
▼	A	22
arccos	8	23
+	E	24
×	·	25
(	6	26
rcl	5	27
×	·	28
÷	G	29
$\sqrt{x}$	1	30
rcl	5	31
)	6	32
=	–	33
stop	0	34
=	–	35

DETERMINANTS

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$$

Execution:

a_1 / RUN / b_1 / RUN / a_2 / RUN / b_2 / RUN / det

sto	2	00
stop	0	01
X	·	02
stop	0	03
—	F	04
(	6	05
rcl	5	06
X	·	07
stop	0	08
)	6	09
—	F	10
=	—	11
stop	0	12
▲	A	13
goto	2	14
0	0	15
0	0	16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

MATRIX MANIPULATION

1. Matrix multiplication (steps 00–11)

$AB = C$

$$C_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$$

Execution:

a_{i1} / RUN / b_{1j} / RUN /
a_{i2} / RUN / b_{2j} / RUN / · · ·
a_{in} / RUN / b_{nj} / RUN / c_{ij}

To restore zero total for next calculation, press C/CE.

Error correction:

For a_{ik} : C/CE / + / a_{ik}
For b_{kj} : ▲▼ /) / C/CE / + / ▲▼ / (/ b_{kj}

2. Back substitution (steps 00–21)

(for $AX = B$ where A is upper triangular)

$$x_{ij} = \frac{\left(b_{ij} - \sum_{k=i+1}^{n} a_{ik} x_{kj}\right)}{a_{ij}}$$

Pre-execution:

▲▼ / ▲▼ / goto / 0 / 0 / for each x_{ij}

sto	2	00
(	6	01
stop	0	02
×	·	03
rcl	5	04
)	6	05
+	E	06
stop	0	07
▼	A	08
goto	2	09
0	0	10
0	0	11
#	3	12
0	0	13
–	F	14
stop	0	15
–	F	16
÷	G	17
stop	0	18
=	–	19
sto	2	20
stop	0	21
×	·	22
rcl	5	23
+	E	24
stop	0	25
=	–	26
▼	A	27
goto	2	28
2	2	29
1	1	30
		31
		32
		33
		34
		35

Execution:

x_{nj} / RUN / a_{in} / RUN / · · · / $x_{i+1,j}$ / RUN / $a_{i,i+1}$ / RUN / $\Sigma a_{ik} x_{kj}$

▲▼ / ▲▼ / goto / 1 / 2 / RUN / b_{ij} / RUN / a_{ii} / RUN / x_{ij}

Error correction:

For x_{kj}: C/CE / + / x_{kj}

For a_{ik}: / ▲▼ /) / C/CE / + / ▲▼ / (/ a_{ik}

For b_{ij}: C/CE / – / b_{ij}

For a_{ii}: C/CE / ÷ / a_{ii}

3. Adding a multiple of row i to row j in the augmented matrix (A/B) (steps 16–30)

$a'_{jk} = a_{jk} + m_{ji} a_{ik}$, $\qquad b'_{jk} = b_{jk} + m_{ji} b_{ik}$

where $m_{ji} = -\dfrac{a_{ji}}{a_{ii}}$

Pre-execution (each m_{ji}):

▲▼ / ▲▼ / goto / 1 / 6 / C/CE

Execution:

a_{ji} / RUN / a_{ii} / RUN / m_{ji} error correction: re-run from 16

a_{ik} / RUN / a_{jk} / RUN / a'_{jk} for each k

b_{ik} / RUN / b_{jk} / RUN / b'_{jk} for each k

Note: If m_{ji} is known pre-execution can be ▲▼ / ▲▼ / goto / 1 / 9 / C/CE and first part of execution m_{ji} / RUN / –

EQUATION SOLVING

The secant method

In this variant of the Newton-Raphson method for solving the equation $f(x) = 0$, instead of computing the derivative $f'(x)$ at each stage, an approximation to $f'(x)$ at a point in the vicinity of a root x_r is used.

Stage 1:

Write a program segment to compute $f(x)$ when x is in memory, taking up no more than 27 steps excluding the final / stop /. Enter the program starting at step 01, ending with the sequence / stop / ▼/ goto / 0 / 0 /.

Execution: x / RUN / $f(x)$

Evaluate $f(x)$ for a range of values in which a root is likely to occur. If $f(x_1)$ and $f(x_2)$ have opposite signs, there is a root between x_1 and x_2.

Stage 2:

Calculate an approximation to the derivative of $f(x)$ as follows:

$f(x_2)$ / – / $f(x_1)$ / ÷ / ▲▼ / (/ x_1 / – / x_2 / ▲▼ /) / = / k $\triangleq -f'(x_r)$

Stage 3:

The iteration formula for the secant method is

$$x' = x + \frac{f(x)}{K}$$

where K is a constant approximately equal to the derivative of $f(x)$ at the root. K may be chosen to be equal to k, or may be an integer or a number with fewer digits than k, in which case it should be numerically larger than k.

Note: If the program segment in Stage 1 took 27 steps, there is room for only one digit for K in the following program. (contd. over)

sto	2	00
		01
		02
		03
		04
		05
		06
		07
		08
		09
		10
		11
		12
		13
		14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
(÷)	(F)	28
÷	G	29
#	3	30
K	K	31
+	E	32
rcl	5	33
=	–	34
stop	0	35

Starting at the final / stop / step, press / ▲▼ / LEARN / and enter the sequence:

/ ÷ / # / K / + / rcl / = / stop /   for K positive, or

/ – / ÷ / # / K / + / rcl / = / stop /   for negative K

The sequence / ▼ / goto / 0 / 0 / or / = / steps may be added at the end.

Execution: x / RUN /
/ RUN / · · ·

Repeat until successive values are equal. If convergence is slow, decrease K. If the results diverge, increase K.

If k is a small fraction, the / ÷ / step may be replaced by a / X / step and K taken as the reciprocal of k.

See below for example.

sto	2	00
cos	8	01
–	F	02
rcl	5	03
÷	G	04
#	3	05
2	2	06
+	E	07
rcl	5	08
=	–	09
stop	0	10
▼	A	11
goto	2	12
0	0	13
0	0	14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

Example:

To solve $\cos x = x$

$f(x) = \cos x - x$

Take $x_1 = \frac{\pi}{2}$, $x_0 = 0$.

$$\text{Then} \quad \frac{f(x_0) - f(x_1)}{x_1 - x_0} = \frac{1 + \frac{\pi}{2}}{\frac{\pi}{2}} = 2$$

Program segment is / cos / – / rcl

Guess 1 as starting solution

Execution:

1 / RUN / [illegible]
/ RUN / 0·7440342
/ RUN / 0·7399375
/ RUN / 0·7392705
/ RUN / 0·7391738
/ RUN / [illegible]
/ RUN / 0·7391519
/ RUN / 0·7391483
/ RUN / [illegible]
/ RUN / 0·7391456
/ RUN / 0·7391451
/ RUN / 0·7391449
/ RUN / 0·7391448
/ RUN / 0·7391447
/ RUN / 0·7391447

So result is 0·7391447

CIRCLES

Circumference and area

X	·	00
(	6	01
X	·	02
#	3	03
6	6	04
·	A	05
2	2	06
8	8	07
3	3	08
1	1	09
9	9	10
=	–	11
stop	0	12
)	6	13
÷	G	14
#	3	15
2	2	16
=	–	17
stop	0	18
▼	A	19
goto	2	20
0	0	21
0	0	22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

Execution:

radius / RUN / circumference / RUN / area

CIRCLES

Radius of circle from area

$$r = \sqrt{\frac{A}{\pi}}$$

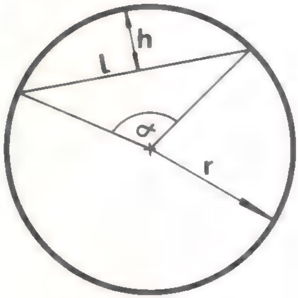
Execution:

A / RUN /

÷	G	00
#	3	01
3	3	02
·	A	03
1	1	04
4	4	05
1	1	06
5	5	07
9	9	08
2	2	09
6	6	10
=	–	11
$\sqrt{x}$	1	12
stop	0	13
▼	A	14
goto	2	15
0	0	16
0	0	17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

CIRCLES

Area of segment:



Area of segment if h and r are given:

$$\text{Area} = \frac{r^2}{2}(\alpha - \sin\alpha)$$

$$\text{where } \cos\frac{\alpha}{2} = \frac{r-h}{r}$$

Note: the angle α is calculated internally and is not required to be input.

Execution:

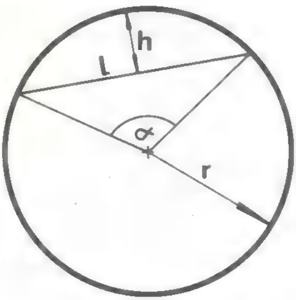
r / RUN / h / RUN / area

Note: limited range, $\alpha < 1{\cdot}57$ radians

sto	2	00
—	F	01
stop	0	02
÷	G	03
rcl	5	04
=	—	05
▼	A	06
arccos	8	07
+	E	08
—	F	09
(	6	10
sin	7	11
)	6	12
×	·	13
(	6	14
rcl	5	15
×	·	16
)	6	17
÷	G	18
#	3	19
2	2	20
=	—	21
stop	0	22
▼	A	23
goto	2	24
0	0	25
0	0	26
		27
		28
		29
		30
		31
		32
		33
		34
		35

CIRCLES

Length of chord



$$l = 2\sqrt{2hr - h^2}$$

Execution:

h / RUN / r / RUN / length

sto	2	00
X	·	01
(	6	02
stop	0	03
+	E	04
–	F	05
rcl	5	06
)	6	07
+	E	08
$\sqrt{x}$	1	09
=	–	10
stop	0	11
▼	A	12
goto	2	13
0	0	14
0	0	15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
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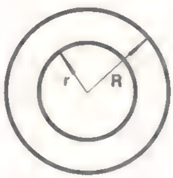
		31
		32
		33
		34
		35

CIRCLES

Area of circular annulus



Area = $\pi(R^2 - r^2)$

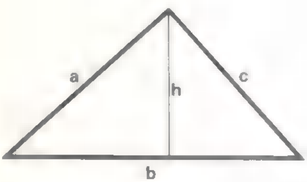
Execution:

R / RUN / r / RUN / area

×	·	00
–	F	01
(	6	02
stop	0	03
×	·	04
)	6	05
×	·	06
#	3	07
3	3	08
·	A	09
1	1	10
4	4	11
1	1	12
5	5	13
9	9	14
=	–	15
stop	0	16
▼	A	17
goto	2	18
0	0	19
0	0	20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

TRIANGLES

To find area, given base and height



$$A = \frac{bh}{2}$$

Execution:

b / RUN / h / RUN / area

X	·	00
stop	0	01
÷	G	02
#	3	03
2	2	04
=	–	05
stop	0	06
▼	A	07
goto	2	08
0	0	09
0	0	10
		11
		12
		13
		14
		15
		16
		17
		18
		19
		20
		21
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		25
		26
		27
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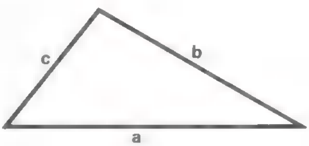
		29
		30
		31
		32
		33
		34
		35

TRIANGLES

To find area, given all three sides



$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where } s = \left(\frac{a+b+c}{2}\right)$$

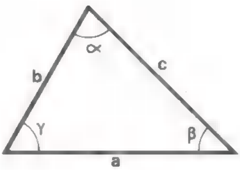
Execution:

a / RUN / b / RUN / c / RUN / b / RUN / a / RUN / area

+	E	00
stop	0	01
+	E	02
stop	0	03
sto	2	04
÷	G	05
#	3	06
2	2	07
X	·	08
(	6	09
▼	A	10
MEx	5	11
–	F	12
rcl	5	13
–	F	14
)	6	15
X	·	16
(	6	17
rcl	5	18
–	F	19
stop	0	20
)	6	21
X	·	22
(	6	23
rcl	5	24
–	F	25
stop	0	26
)	6	27
=	–	28
$\sqrt{x}$	1	29
stop	0	30
▼	A	31
goto	2	32
0	0	33
0	0	34
		35

TRIANGLES

Finding a side, given two angles and a side



$$a = \frac{b \sin \alpha}{\sin \beta}$$

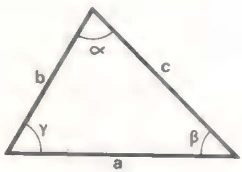
Execution:

$\alpha°$ / RUN / $\beta°$ / RUN / b / RUN /

–	F	00
#	3	01
9	9	02
0	0	03
X	·	04
=	–	05
$\sqrt{x}$	1	06
▼	A	07
D→R	3	08
cos	8	09
÷	G	10
(	6	11
stop	0	12
–	F	13
#	3	14
9	9	15
0	0	16
X	·	17
=	–	18
$\sqrt{x}$	1	19
cos	8	20
)	6	21
X	·	22
stop	0	23
=	–	24
stop	0	25
▼	A	26
goto	2	27
0	0	28
0	0	29
		30
		31
		32
		33
		34
		35

TRIANGLES

Length of third side from two sides and included angle



$$a = \sqrt{b^2 + c^2 - 2bc\cos\alpha}$$

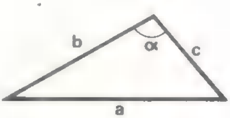
Execution:

b / RUN / c / RUN / α° / RUN / a

sto	2	00
stop	0	01
X	·	02
(	6	03
–	F	04
rcl	5	05
X	·	06
=	–	07
▼	A	08
MEx	5	09
+	E	10
)	6	11
X	·	12
(	6	13
stop	0	14
–	F	15
#	3	16
9	9	17
0	0	18
=	–	19
▼	A	20
D→R	3	21
sin	7	22
+	E	23
#	3	24
1	1	25
=	–	26
)	6	27
+	E	28
rcl	5	29
=	–	30
$\sqrt{x}$	1	31
stop	0	32
=	–	33
=	–	34
=	–	35

TRIANGLES

Finding an angle, given three sides



$$\cos \alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

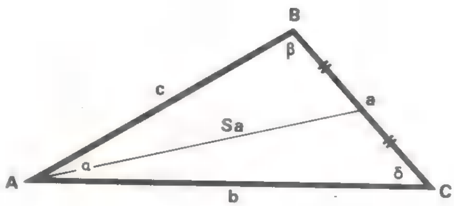
Execution:

a / RUN / b / RUN / c / RUN /

÷	G	00
stop	0	01
sto	2	02
X.	·	03
–	F	04
+	E	05
#	3	06
1	1	07
X	·	08
(	6	09
stop	0	10
÷	G	11
rcl	5	12
=	–	13
sto	2	14
÷	G	15
)	6	16
+	E	17
rcl	5	18
÷	G	19
#	3	20
2	2	21
=	–	22
▼	A	23
arcsin	7	24
▼	A	25
R→D	6	26
–	F	27
+	E	28
#	3	29
9	9	30
0	0	31
=	–	32
stop	0	33
=	–	34
=	–	35

TRIANGLES

Length of medians, given lengths of sides



$$S_a = \frac{\sqrt{2(b^2 + c^2) - a^2}}{2}$$

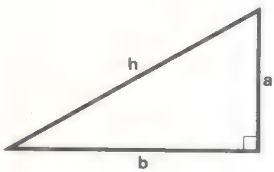
Execution:

b / RUN / c / RUN / a / RUN / S_a

×	·	00
+	E	01
(	6	02
stop	0	03
×	·	04
)	6	05
+	E	06
−	F	07
(	6	08
stop	0	09
×	·	10
)	6	11
÷	G	12
#	3	13
4	4	14
=	−	15
$\sqrt{x}$	1	16
stop	0	17
▼	A	18
goto	2	19
0	0	20
0	0	21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

RIGHT ANGLED TRIANGLES

Length of hypotenuse from other two sides



$$h = \sqrt{a^2 + b^2}$$

Execution:

a / RUN / b / RUN /

X	·	00
+	E	01
(	6	02
stop	0	03
X	·	04
)	6	05
=	–	06
$\sqrt{x}$	1	07
stop	0	08
▼	A	09
goto	2	10
0	0	11
0	0	12
		13
		14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
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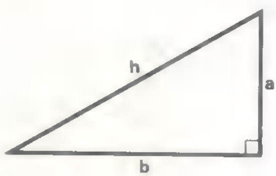
		31
		32
		33
		34
		35

RIGHT ANGLED TRIANGLES

Length of one short side from other two sides



$$b = \sqrt{h^2 - a^2}$$

Execution:

a / RUN / h / RUN / b

X	·	00
–	F	01
(	6	02
stop	0	03
X	·	04
)	6	05
–	F	06
=	–	07
$\sqrt{x}$	1	08
stop	0	09
▲	A	10
goto	2	11
0	0	12
0	0	13
		14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
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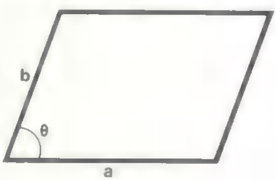
		31
		32
		33
		34
		35

PARALLELOGRAMS



Area = ab sin θ

Execution:

a / RUN / b / RUN / $\theta°$ / RUN / area

For θ in radians, insert / ▼ / R→D / between steps 04 and 05.

X	·	00
stop	0	01
X	·	02
(	6	03
stop	0	04
–	F	05
#	3	06
9	9	07
0	0	08
=	–	09
▼	A	10
D→R	3	11
cos	8	12
)	6	13
=	–	14
stop	0	15
▼	A	16
goto	2	17
0	0	18
0	0	19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

SPHERES

Surface area and volume

$A = 4\pi r^2$

$V = \frac{4}{3}\pi r^3$

Execution:

radius / RUN / surface area / RUN / volume

X	·	00
(	6	01
X	·	02
X	·	03
#	3	04
1	1	05
2	2	06
·	A	07
5	5	08
6	6	09
6	6	10
3	3	11
7	7	12
1	1	13
=	–	14
stop	0	15
)	6	16
÷	G	17
#	3	18
3	3	19
=	–	20
stop	0	21
▼	A	22
goto	2	23
0	0	24
0	0	25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

SPHERES

Radius from volume

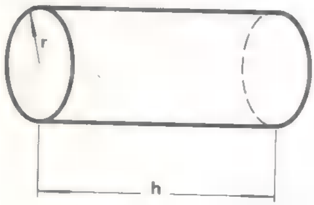
$$r = \sqrt[3]{\frac{3V}{4\pi}}$$

Execution:

V / RUN / r

X	·	00
#	3	01
·	A	02
2	2	03
3	3	04
8	8	05
7	7	06
3	3	07
2	2	08
4	4	09
=	–	10
ln	4	11
÷	G	12
#	3	13
3	3	14
=	–	15
▼	A	16
e^x	4	17
stop	0	18
▼	A	19
goto	2	20
0	0	21
0	0	22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

CYLINDERS



sto	2	00
X	·	01
X	·	02
#	3	03
6	6	04
·	A	05
2	2	06
8	8	07
3	3	08
1	1	09
8	8	10
5	5	11
3	3	12
+	E	13
(	6	14
÷	G	15
#	3	16
2	2	17
X	·	18
stop	0	19
÷	G	20
stop	0	21
rcl	5	22
+	E	23
)	6	24
stop	0	25
=	–	26
stop	0	27
▼	A	28
goto	2	29
0	0	30
0	0	31
		32
		33
		34
		35

Volume = $\pi r^2 h$

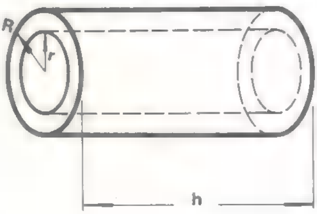
Area of curved surface = $2\pi rh$

Total surface area = $2\pi r(r + h)$

Execution:

r / RUN / h / RUN / volume / RUN / area of curved surface / RUN / total surface area

HOLLOW CYLINDRICAL TUBE



Area of curved surface = $2\pi h(R + r)$

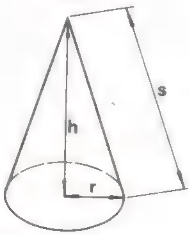
Volume = $\pi h(R^2 - r^2)$

Execution:

R / RUN / r / RUN / h / RUN / area of curved surface / RUN / volume

+	E	00
stop	0	01
sto	2	02
÷	G	03
#	3	04
2	2	05
–	F	06
▼	A	07
MEx	5	08
=	–	09
▼	A	10
MEx	5	11
X	·	12
stop	0	13
X	·	14
#	3	15
1	1	16
2	2	17
·	A	18
5	5	19
6	6	20
6	6	21
X	·	22
stop	0	23
rcl	5	24
=	–	25
stop	0	26
▼	A	27
goto	2	28
0	0	29
0	0	30
		31
		32
		33
		34
		35

RIGHT CIRCULAR CONE



$$\text{Volume} = \frac{\pi r^2 h}{3}$$

$$\text{Curved surface area} = \pi r\sqrt{r^2 + h^2}$$

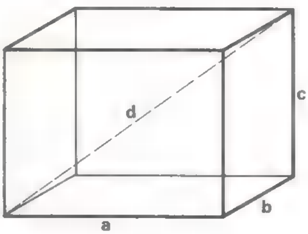
$$\text{Total surface area} = \pi r\left(r + \sqrt{r^2 + h^2}\right)$$

Execution:

h / RUN / r / RUN / area of curved surface / ▲▼ / rcl / area of base / RUN / total surface area / RUN / volume

X	·	00
(	6	01
÷	G	02
stop	0	03
sto	2	04
X	·	05
+	E	06
#	3	07
1	1	08
=	–	09
$\sqrt{x}$	1	10
▼	A	11
MEx	5	12
X	·	13
X	·	14
#	3	15
3	3	16
·	A	17
1	1	18
4	4	19
1	1	20
6	6	21
X	·	22
▼	A	23
MEx	5	24
+	E	25
stop	0	26
=	–	27
stop	0	28
rcl	5	29
)	6	30
÷	G	31
#	3	32
3	3	33
=	–	34
stop	0	35

RECTANGULAR PARALLELEPIPED



Diagonal:

$$d = \sqrt{a^2 + b^2 + c^2}$$

Execution:

a / RUN / b / RUN / c / RUN / d

×	·	00
+	E	01
(	6	02
stop	0	03
×	·	04
)	6	05
+	E	06
(	6	07
stop	0	08
×	·	09
)	6	10
=	–	11
$\sqrt{x}$	1	12
stop	0	13
▼	A	14
goto	2	15
0	0	16
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RECTANGULAR PARALLELEPIPED

Surface area

$A = 2(ab + ac + bc)$

Execution:

a / RUN / b / RUN / c / RUN / area

sto	2	00
stop	0	01
+	E	02
(	6	03
X	·	04
rcl	5	05
=	–	06
▼	A	07
MEx	5	08
)	6	09
X	·	10
stop	0	11
+	E	12
rcl	5	13
+	E	14
=	–	15
stop	0	16
▼	A	17
goto	2	18
0	0	19
0	0	20
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DISTANCE BETWEEN TWO POINTS IN SPACE

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

points are (x_1, y_1, z_1) and (x_2, y_2, z_2)

Execution:

x_1 / RUN / x_2 / RUN / y_1 / RUN / y_2 / RUN / z_1 / RUN / z_2 / RUN /

–	F	00
stop	0	01
X	·	02
+	E	03
(	6	04
stop	0	05
–	F	06
stop	0	07
X	·	08
)	6	09
+	E	10
(	6	11
stop	0	12
–	F	13
stop	0	14
X	·	15
)	6	16
=	–	17
$\sqrt{x}$	1	18
stop	0	19
▼	A	20
goto	2	21
0	0	22
0	0	23
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COORDINATE CONVERSION

Polar to cartesian

θ in radians, $-\pi < \theta < \pi, \theta \neq 0$

Execution:

r / RUN / θ / RUN / [illegible] / RUN / y

If $\theta = 0$, x = r and y = 0

If $\theta = \pi$, x = −r and y = 0

×	·	00
(	6	01
stop	0	02
÷	G	03
#	3	04
2	2	05
=	–	06
tan	9	07
sto	2	08
÷	G	09
+	E	10
rcl	5	11
÷	G	12
+	E	13
)	6	14
=	–	15
▼	A	16
MEx	5	17
–	F	18
(	6	19
÷	G	20
)	6	21
÷	G	22
#	3	23
2	2	24
–	F	25
×	·	26
rcl	5	27
=	–	28
stop	0	29
rcl	5	30
stop	0	31
=	–	32
=	–	33
=	–	34
=	–	35

COORDINATE CONVERSION

Cartesian to polar

÷	G	00
(	6	01
X	·	02
÷	G	03
stop	0	04
sto	2	05
+	E	06
rcl	5	07
X	·	08
rcl	5	09
=	–	10
$\sqrt{x}$	1	11
stop	0	12
)	6	13
+	E	14
#	3	15
1	1	16
÷	G	17
#	3	18
2	2	19
=	–	20
$\sqrt{x}$	1	21
▼	A	22
arccos	8	23
+	E	24
X	·	25
(	6	26
rcl	5	27
X	·	28
÷	G	29
$\sqrt{x}$	1	30
rcl	5	31
)	6	32
=	–	33
stop	0	34
=	–	35

Restriction: $y \neq 0$

If $y = 0$, $r = |x|$

and $\theta = 0$ if $x \geqslant 0$

π if $x < 0$

Execution:

x / RUN / y / RUN / / RUN /

RADIUS OF CURVATURE

×	·	00
+	E	01
#	3	02
1	1	03
×	·	04
(	6	05
$\sqrt{x}$	1	06
)	6	07
÷	G	08
stop	0	09
=	–	10
stop	0	11
▼	A	12
goto	2	13
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$$r = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$$

Execution:

$/ \frac{dy}{dx} /$ RUN $/ \frac{d^2y}{dx^2} /$ RUN / r

HAVERSINE AND INVERSE HAVERSINE, VERSINE AND SUVERSINE

Haversine:

pre-execution: ▲▼ / ▲▼ / goto / 0 / 0 /

Execution:

$\theta°$ / RUN / hav θ

/ + / = / vers θ / – / + / 2 / = / suvers θ

Inverse haversine:

pre-execution: ▲▼ / ▲▼ / goto / 1 / 4 /

Execution:

hav θ / RUN / $\theta°$

vers θ / ÷ / 2 / = / RUN / $\theta°$

suvers θ / – / + / 2 / ÷ / 2 / = / RUN / $\theta°$

Range $0 \leqslant \theta° \leqslant 180$

For vers θ see post and pre-execution.

▼	A	00
D→R	3	01
÷	G	02
#	3	03
2	2	04
=	–	05
sin	7	06
X	·	07
=	–	08
stop	0	09
▼	A	10
goto	2	11
0	0	12
0	0	13
$\sqrt{x}$	1	14
▼	A	15
arcsin	7	16
+	E	17
=	–	18
▼	A	19
R→D	6	20
stop	0	21
▼	A	22
goto	2	23
1	1	24
4	4	25
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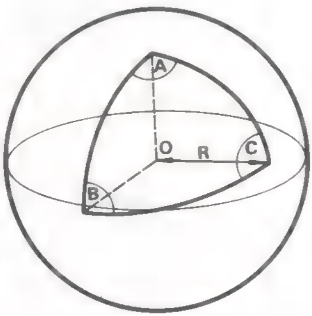
AREA OF A SPHERICAL TRIANGLE



Area = $(A + B + C - \pi)R^2$

A, B, C in degrees

Execution:

A / RUN / B / RUN / C / RUN / R / RUN / STEP

+	E	00
stop	0	01
+	E	02
stop	0	03
–	F	04
#	3	05
1	1	06
8	8	07
0	0	08
=	–	09
▼	A	10
D→R	3	11
X	·	12
(	6	13
stop	0	14
X	·	15
)	6	16
=	–	17
stop	0	18
▼	A	19
goto	2	20
0	0	21
0	0	22
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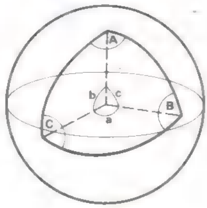
SPHERICAL TRIANGLES: SINE RULE

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$



Angles in degrees

Execution:

a / RUN / A / RUN / B / RUN / b / C / RUN / c

or

A / RUN / a / RUN / b / RUN / B / c / RUN / C

Note: If a result of 0 appears, the final arcsin had an out-of-range argument and the result is impossible for the particular angles given, or else very close to 90°.

For angle $A > 90°$, compute using 180 / – / A / = / etc.

Special execution: navigation

To find course from place 2 to place 1

$$\sin C = \frac{\sin (E_1 - E_2) \cos N_2}{\sin d}$$

Execution:

E_1 / – / E_2 / RUN / d / RUN / 90 / – / N_2 / = / RUN / C

where E_1 = easterly longitude of place 1
E_2 = easterly longitude of place 2
N_2 = north latitude of place 2
d = angular distance between places 1 and 2

(For westerly longitudes or south latitudes, change sign of angle.)

▼	A	00
D→R	3	01
sin	7	02
÷	G	03
(	6	04
stop	0	05
▼	A	06
D→R	3	07
sin	7	08
)	6	09
X	·	10
sto	2	11
(	6	12
stop	0	13
▼	A	14
D→R	3	15
sin	7	16
)	6	17
=	–	18
▼	A	19
arcsin	7	20
▼	A	21
R→D	6	22
stop	0	23
▼	A	24
D→R	3	25
sin	7	26
X	·	27
rcl	5	28
▼	A	29
goto	2	30
1	1	31
8	8	32
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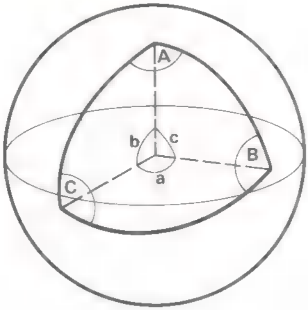
SPHERICAL TRIANGLES:

Cosine Rule



$\cos a = \cos b\,(\cos c + \sin c \tan b \cos A)$

Execution:

c / RUN / A / RUN / b / RUN / b / RUN / a

Navigation

To find great circle distance between places 1 and 2

1. Latitude N_1 longitude E_1 (–ve if W)
2. Latitude N_2 longitude E_2 (–ve if W)

Execution:

90 / – / N_2 / = / RUN / E_1 / – / E_2 / RUN / 90 / – / N_1 / RUN / 90 / – / N_1 / RUN / α (degrees)

X / 111·19 / = / distance in km

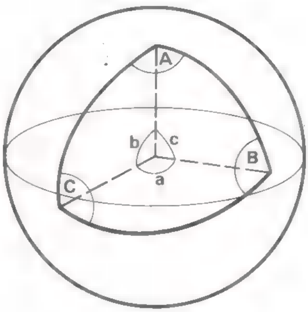
or X / 69·41 / = / distance in miles

For angles greater than 90° use appropriate reductions to first quadrant.

▼	A	00
D→R	3	01
sto	2	02
sin	7	03
X	·	04
(	6	05
stop	0	06
▼	A	07
D→R	3	08
cos	8	09
)	6	10
X	·	11
(	6	12
stop	0	13
▼	A	14
D→R	3	15
tan	9	16
)	6	17
+	E	18
(	6	19
rcl	5	20
cos	8	21
)	6	22
X	·	23
(	6	24
stop	0	25
▼	A	26
D→R	3	27
cos	8	28
)	6	29
=	–	30
▼	A	31
arccos	8	32
▼	A	33
R→D	6	34
stop	0	35

SPHERICAL TRIANGLES

The Cosine Rule – to find an angle or side given three sides or angles



$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}$$

$$\cos a = \frac{\cos A + \cos B \cos C}{\sin B \sin C}$$

Execution:

Angles in radians

c / RUN / b / RUN / – / RUN / a / RUN / A

C / RUN / B / RUN / RUN / A / RUN / a

For angles in degrees use

▲▼ / ▲▼ / D→R / after each angle.

sto	2	00
sin	7	01
▼	A	02
MEx	5	03
cos	8	04
X	·	05
(	6	06
stop	0	07
sin	7	08
X	·	09
▼	A	10
MEx	5	11
=	–	12
▼	A	13
MEx	5	14
X	·	15
–	F	16
#	3	17
1	1	18
–	F	19
)	6	20
stop	0	21
+	E	22
(	6	23
stop	0	24
cos	8	25
)	6	26
÷	G	27
rcl	5	28
=	–	29
▼	A	30
arccos	8	31
stop	0	32
=	–	33
=	–	34
=	–	35

SPHERICAL TRIANGLES:

Half-angle tangent formula

$$\tan\frac{A}{2} = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin(s-a)\sin s}}$$

where $s = \dfrac{a+b+c}{2}$

$$\tan\frac{a}{2} = \sqrt{\frac{\cos(\pi-S)\cos(S-A)}{\cos(S-B)\cos(S-C)}}$$

where $S = \dfrac{A+B+C}{2}$

Execution:

Angles in radians

a / + / b / + / c / RUN / b / RUN / a / RUN /

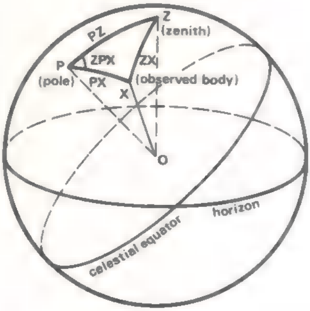
$\frac{A}{2}$ / = / A

For the cosine version change all / sin / steps to / cos /.

sto	2	00
÷	G	01
#	3	02
2	2	03
—	F	04
▼	A	05
MEx	5	06
=	—	07
sin	7	08
÷	G	09
(	6	10
rcl	5	11
sin	7	12
)	6	13
X	·	14
(	6	15
stop	0	16
—	F	17
rcl	5	18
=	—	19
sin	7	20
)	6	21
÷	G	22
(	6	23
stop	0	24
—	F	25
rcl	5	26
=	—	27
sin	7	28
)	6	29
=	—	30
$\sqrt{x}$	1	31
▼	A	32
arctan	9	33
+	E	34
stop	0	35

SPHERICAL TRIANGLES

Solving the PZX triangle



$$\text{hav } ZX = \text{hav } (PX \sim PZ) + \sin PX \sin PZ \text{ hav } \angle ZPX$$
$$= \text{hav } (L \sim D) + \cos L \cos D \text{ hav } \angle ZPX$$

(for the second formula use cos at steps 10 and 27 instead of sin)

ZX is the calculated zenith distance (CZD)

Enter south latitudes as –ve

Execution:

Angles in radians –

∠ZPX / RUN / PX / RUN / PZ / RUN / + / = / ZX

Angles in degrees –

∠ZPX° / ▲▼ / ▲▼ / D→R / RUN / PX° / ▲▼ / ▲▼ / D→R / RUN / PZ° / ▲▼ / ▲▼ / D→R / RUN / + / = / ▲▼ / ▲▼ / R→D / ZX°

Intercept I = CZD – TZD
(calculated – true zenith distance)

Post-execution:

/ – / TZD / X / 60 / = / I

(I in minutes of arc or miles approx.)

÷	G	00
+	E	01
÷	G	02
=	–	03
sin	7	04
X	·	05
X	·	06
(	6	07
stop	0	08
sto	2	09
sin	7	10
)	6	11
X	·	12
(	6	13
stop	0	14
–	F	15
▼	A	16
MEx	5	17
÷	G	18
#	3	19
2	2	20
=	–	21
sin	7	22
X	·	23
=	–	24
▼	A	25
MEx	5	26
sin	7	27
)	6	28
+	E	29
rcl	5	30
=	–	31
$\sqrt{x}$	1	32
▼	A	33
arcsin	7	34
stop	0	35

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